MATH 20D: Final Exam Practice Review Problems

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Remember to list the sources you used when completing the assignment.
Below *NSS* is used to reference the text *Fundamentals of Differential Equations (9th edition) by Nagle, Saff, Snider*

Question (1)**.** (NSS 2.2.17 & 25)
Using separation of variables give a general solution to each of the ODE's below. Then find a solution to the ODE satisfying the given initial condition

(a) $\frac{dy}{dx} = x^2(1+y), \quad y(0) = 3$ ***(b)*** $\frac{dy}{dx} = (1+y^2)\tan(x), \quad y(0) = \sqrt{3}$

Question (2)**.** (NSS 2.3.17 & 22)
Using the method of integrating factors give a general solution to each of the ODE's below. Then find a solution to the ODE satisfying the given initial condition

(a) $\frac{dy}{dx} - \frac{y}{x} = xe^x, \quad y(1) = e - 1$ ***(b)*** $\sin(x)\frac{dy}{dx} + y\cos(x) = x\sin(x), \quad y\left(\frac{\pi}{2}\right) = 2$

Question (3)**.** (NSS 3.3.3) *A bottle of white wine of temperature 70 °F is placed in a portable cooler which has temperature 32 °. If it takes 15 min for the wine to chill to 60 °F use Newton's law of cooling to predict how long will it take for the wine to reach 56 °F?*

Question (4)**.** (NSS 4.2.13 & 17 and 4.3.21)
For each of the ODE's below, write down a general solution to the equation, then solve the ODE for the given intial conditions

(a) $y'' + 2y' - 8y = 0; \quad y(0) = 3, \quad y'(0) = -12,$

(b) $y'' - 6y' + 9y = 0; \quad y(0) = 2, \quad y'(0) = 25/3$

(c) $y'' + 2y' + 2y = 0; \quad y(0) = 2, \quad y'(0) = 1$

Question (5)**.** (NSS 4.9.11) *A 1 kilogram mass is attached to a spring with stiffness 100 N/m. The damping constant for the system is 0.2 N-sec/m. If the mass is pushed rightward from the equilibrium position with velocity of 1 m/sec, when will it attain its maximum dispacement to the right?*

Question (6)**.** (NSS 4.6.3,5, & 23)
(a) *Using the method of variation of parameters find a general solution to each of the ODE's below*

(i) $y'' - 2y' + y = t^{-1}e^t$ ***(ii)*** $y''(\theta) + 16y(\theta) = \sin(4\theta)$

(b) Given that $y_1(t) = e^t$ and $y_2(t) = t+1$ are linearly indepedendent solutions to the homogeneous equation

$$ty'' - (t+1)y' + y = 0$$

find a particular solution to the equation $ty'' - (t+1)y' + y = t^2$.

Question (7)**.** (NSS 4.7.26,31, & 41)

(a) Let $y_1(t) = t^3$ and $y_2(t) = |t|^3$. Determine whether $y_1(t)$ and $y_2(t)$ are linearly independent over the following intervals

$$\textbf{(a)}\ [0,\infty) \qquad \textbf{(b)}\ (-\infty,0] \qquad \textbf{(c)}\ (-\infty,\infty)$$

Compute the Wronskian $W[y_1, y_2](t)$ on the interval $(-\infty,\infty)$.

(b) Determine which of the follows functions can occur as the Wronskian on $-1 < t < 1$ for a pair of solutions to some equation $y''(t) + p(t)y'(t) + q(t)y(t) = 0$ with $p(t)$ and $q(t)$ continuous on $(-1, 1)$.

$$\textbf{(a)}\quad w(t) = 6e^{4t} \qquad \textbf{(b)}\quad w(t) = t^3$$

$$\textbf{(c)}\quad w(t) = (t+1)^{-1} \qquad \textbf{(d)}\quad w(t) \equiv 0.$$

(c) Given that $f(t) = t^{-1}$ solves the ODE

$$y'' - 2ty' - 4y = 0, \qquad t > 0,$$

use the method of reduction of order to find a second linearly independent solution.

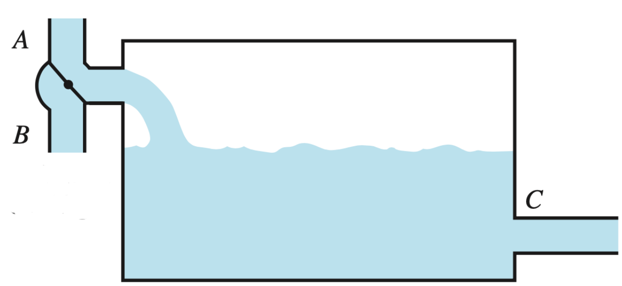
Question (8)**.** (NSS 7.5.3, 7.6.21, & 7.9.14) *Solve each of the initial value problems below using the method of Laplace transform*

$$\textbf{(a)}\quad y'' + 6y' + 9y = 0; \qquad y(0) = -1, \quad y'(0) = 6$$

$$\textbf{(b)}\quad y'' + y = u(t-3); \qquad y(0) = 0, \quad y'(0) = 1$$

$$\textbf{(c)}\quad y'' + 2y' + 2y = \delta(t-\pi); \quad y(0) = 1, \quad y('(0) = 1.$$

Question (9)**.** *The mixing tank in the figure below holds 250L of a brine solution with a salt concentration of 0.04kg/L. For the first 10 minutes of operation, valve A is open adding 6L/min of a brine containing 0.08 kg/L salt concentration. After 10 minutes, valve B is switched in, adding 0.06kg/L concentration at 6L/min. The exit valve C removes 6 L/min, thereby keeping the volume constant. FInd the concentration of salt in the tank as a function of time*



Question (10)**.** (NSS 9.4.5,7) *Rewrite each of the scalar differential equations below as a first order matrix differential equation in normal form*

(a) $y'''(t) - 3y'(t) - 10y(t) = \sin(t)$ ***(b)*** $\frac{d^4w}{dt^4} + w = t^2$.

Question (11)**.** (NSS Chapter 9 Review Problems 1 & 2) *For each of the matrices* A *below find a general solution for the system* $\mathbf{x}'(t) = A\mathbf{x}(t)$ *where* $\mathbf{x}(t) = \mathrm{col}(x_1(t), x_2(t))$

(a) $A = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$ ***(b)*** $A = \begin{pmatrix} 3 & 2 \\ -5 & 1 \end{pmatrix}$ ***(c)*** $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$